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Magnetized Two-Fluid Spin Quantum Plasmas and Impurity Effect

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Abstract: In an electron-ion plasma, ions can consider are fixed and electrons moving due to the high mass of ions relative to electrons. In a piece of metal, free electrons are almost like electrons in a plasma, and ions are stationary. By applying electric and magnetic fields, the behavior of these electrons can be predicted by studing the two-fluid electron-ion model. This paper derives a set of two-fluid (electron-ion) plasma equations based on the quantum magnetic hydrodynamic model (QMHD) for each of the two electron-ion fluids. We consider the electron-ion as two different types of particles and follow a path for discussion that is different from the usual path and obtain new dispersion equations. We consider the two regimes of non-spin and spin plasma separately and analyze the propagation of waves that correspond to perturbations in parallel and perpendicular to the external magnetic field, and obtain their vibrational modes. Then we return to the subject of the metal part and the ions and set the flow velocity of the ions to zero. Finally, we consider a one-dimensional grid of ions, at any given length L_0, with one electron impurity as a Fermi polaron. We study its effect on ground state energy. Due to the long-range nature of the electron-ion interaction, these systems have several properties distinct from their ordinary counterparts such as the simultaneous presence of several stable. Surprisingly, the residue of electrons is shown to increase with the Fermi density for fixed interaction strength.

Keywords: Fermi Polaron, Magnetized Two-Fluid Spin Quantum Plasmas, Quantum Hydrodynamic Plasmas, Spin-Spin Interaction, Spin-Magnetic Field Coupling

1. Introduction

Magneto-hydrodynamics (MHD) can consider a suitable formalism for studying magnetized plasma at scales larger than the ionic inertia length $\lambda_i = c/\omega_i$ where ω_i is the ionic plasma frequency [1-4]. At distances much smaller than λ_{ion}, we assume, ions are stationary due to their much larger mass than electrons, and electric currents are entirely the result of the motion of electrons. At these scales, quantum works are highlighted. One of these is when the thermal de Broglie wavelength of the plasma particles $\lambda_B = \hbar/\sqrt{k_B T m}$ is about the average particle distance $\bar{L} \equiv n_0^{-1/3}$, i.e., $\lambda_B \gtrsim \bar{L}$. A method for considering the quantum effect is to correct the classical equations. It is natural to see differences between the classical and quantum models, for example, using the quantum hydrodynamic model to see new oscillating modes in a magnetic quantum plasma [5-7].

This work is organized as follows: In Section 2, we use the two-fluid plasma equations and their oscillation modes concerning quantum effects such as Fermi pressure, Bohm pressure, and spin interactions [8-10]. We also get new dispersion modes by converting from the laboratory to the center of the mass system. In Section 3, we study free electrons and ions in a piece of metal. We assume, in the one-dimensional grid of ions, at any given length $\mathcal{L}_0$, there is one electron impurity as a Fermi polaron. We study its effect on ground state energy by eliminating the degree of freedom of the impurity and replacing a generalized Hamiltonian of only ions (impurity removal) with the system's Hamiltonian. Finally, some conclusions are drawn in Section 4.

2. QMHD Equations and the Effects of Spin

We first consider a plasma consisting of two fermionic fluids whose temperature is below the Fermi temperature. Then, we consider electron-ion as two different species and obtain our equations to the evolution of the spin current. We write the QMHD quantum equations for the two-fluid plasma of electron-ion located in the external magnetic field B.

2.1. Two-Fluid Plasma Equations

We use the method of many-particle quantum hydrodynamics for the two-fluid plasma equations. Let's start with the one particle and many particle Pauli equation [11, 12],

$$\hbar\partial_t\psi_\pm(r) = \left[\frac{1}{2m}(p-qm)^2 + q\Phi\right]\hat{I}\psi_\pm(r) - \frac{q\hbar}{2m}\sigma.B\psi_\pm(r) \quad (1)$$

$$i\hbar\partial_t\psi(r_1,\dots,r_N,t) = H\psi(r_1,\dots,r_N,t) \quad (2)$$

Where $\psi_\pm = \begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}$, $\hat{I} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, $\psi_\pm(r)$ is the one-particle wave function, $\psi(r_1,\dots,r_N,t)$ is the many-particle wave function, r_i is the coordinates of each particle and N is the total number of particles in the system. Hamiltonian for electron- ion plasmas of particles moving with velocities $v \ll c$ is [13, 14],

$$H = \left\{\sum_{i=1}^{N}\left[\frac{1}{2m_{e.io}}\left(\hbar\nabla_i \pm \frac{e}{c}A_{i,ext}\right)^2 \mp e\varphi_i^{ext} \pm \frac{e\hbar}{2m_{e.io}}\,\sigma.B_i^{ext}\right] - \frac{1}{2}\sum_{i,j\neq i}^{N}\left[\frac{\pi e^2\hbar^2}{m_{e.io}^2c^2}\sigma_i^e.\sigma_j^{io}\right] + \sum_{i=1}^{N}\frac{e^2\hbar}{4m_{e.io}^2c^2}\sigma.\left(E\times A_{i,ext}\right)\right. \quad (3)$$

Where N_e, N_{io} are numbers of electrons and ions correspondingly, $N = N_e + N_{io}$, φ_i^{ext} is the external scalar potential acting on the particle with number i, $A_{i,ext}$ is the external magnetic potential, σ_i is the Pauli matrixes. Let us describe the physical meaning of different terms in the Hamiltonian (3). The first term describes kinetic energy; The second term is the potential energy of charges in the external electric field. The third term is the Potential energy due to magnetic moments' interaction with the external magnetic field. These three terms are related to the motion of each particle in the external electromagnetic field. The fourth term is the spin-spin interaction. The fifth term is the interaction of the spin-electric-magnetic field correspondingly [15-18].

In a piece of metal, free electrons and ions are almost like electrons and ions in an electrically neutral degenerate plasma composed of two species as a two-fluid electron-ion system. We introduce the velocity for each species of particles $J_{e,io}(r,t) = n_{e,io}(r,t)u_{e,io}(r,t)$, where $u_{e,io}$ is their fluid velocities. In the following, we consider ions to be protons for simplicity. The Fermi pressure for each fluid $P_{e,p}$ is equal to,

$$p_{F,ep} = \frac{2}{5}n_{ep}E_{Fep} = (3\pi^2)^{2/3}\frac{\hbar^2}{5m_{e.p}}n_{ep}^{5/3} \quad (4)$$

where E_{Fep} is the Fermi energy of species e and p, $E_{Fep} = \frac{\hbar^2}{2m}\left(3\pi^2 n_{ep}\right)^{2/3}$. At first, we can write the continuity equation for electrons and protons as,

$$\partial_t n_{e.p} + \nabla.\left(n_{e.p}u_{e.p}\right) = 0 \quad (5)$$

The basic set of equations that we use for the plasma using the quantum hydrodynamic model are as follows [19, 20],

$$m_{e.p}n_{ep}\left(\partial_t + u_{ep}.\nabla\right)u_{ep} + \nabla p_{ep} - \frac{n_{ep}\hbar^2}{2m_{e.p}}\nabla\left(\frac{\nabla^2\sqrt{n_{ep}}}{\sqrt{n_{ep}}}\right) = \mp e n_{ep}\left(E + u_{ep}\times B\right) \mp \frac{e\hbar n_{ep}}{2m_{e.p}c}\left(S_{ep}.\nabla\right)\mathfrak{B}_{ep} + \frac{\hbar^2 n_{ep}}{2m_{e.p}}\nabla\left(\partial_\mu S_\nu^{ep}\,\partial_\mu S_\nu^{ep}\right) \quad (6)$$

And,

$$d_t\, S_p = +\frac{e}{m_p c}S_p \times \mathfrak{B}_p \quad (7)$$

$$d_t\, S_e = -\frac{e}{m_e c}S_e \times \mathfrak{B}_e \quad (8)$$

Where p_{ep} is the partial scalar pressure and the $\mathfrak{B}_p$ is the generalized magnetic field,

$$\mathfrak{B}_p \equiv B + \frac{\hbar}{2en_p}\nabla\left(n_p\nabla.S_p\right) \quad (9)$$

$$\mathfrak{B}_e \equiv B - \frac{\hbar}{2en_e}\nabla\left(n_e\nabla.S_e\right) \quad (10)$$

The Maxwell equations for neutral, non-relativistic systems are,

$$\nabla\times E = -\frac{1}{c}\partial_t B, \nabla\times B = \frac{4\pi e}{c}\left(n_p u_p - n_e u_e\right) + \frac{1}{c}\partial_t E \quad (11)$$

In Eq. (6), the second and third terms on the left show the contribution of pressure to the motion of the particles. The first term on the right represents the Lorentz force; the second term shows spin interaction with the non-uniform magnetic field, and the third term shows spin interaction with non-uniform spin. Now, we convert the two-fluid equations to a single fluid.

2.2. Investigation of Spin Wave Propagation and Vibration Modes

Before starting the discussion, we state some definitions, $m \equiv m_e + m_p$, $\mu \equiv m_e/m$, $m\bar{u} = m_e u_e + m_p u_p$ and $\bar{u} = (1-\mu)u_p + \mu u_e$. Corresponding to the density of the electric current $J = e\left(n_p u_p - n_e u_e\right)$, we also introduce the spin current density $\mathcal{S} = \mathcal{C}\left(n_p u_p - n_e u_e\right)$, where $\mathcal{C} = \frac{e}{m}$, we can conclude,

$$u_e = \bar{u} - \frac{\mathcal{C}(1-\mu)}{n}\mathcal{S} \quad (12)$$

$$u_p = \bar{u} + \frac{\mathcal{C}\mu}{n}\mathcal{S} \quad (13)$$

According to the definition of the mean velocity $\bar{u}$ and the total density n, the continuity equation and equations of motion for each species become,

$$\partial_t n + \nabla.\left(n\bar{u}\right) = 0 \quad (14)$$

$$\mu m d_t u_e + \frac{1}{n}\nabla p_e - \frac{\hbar^2}{2\mu m}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) = -e(E + u_e \times B) - \frac{e\hbar}{2\mu mc} S_\mu^e \nabla \mathfrak{B}_e^\mu + \frac{\hbar^2}{2\mu m}\nabla\left(\partial_\mu S_v^e \partial_\mu S_v^e\right) \tag{15}$$

$$(1-\mu) m d_t u_P + \frac{1}{n}\nabla p_P - \frac{\hbar^2}{2(1-\mu)m}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) = e(E + u_P \times B) + \frac{e\hbar}{2(1-\mu)mc} S_\mu^P \nabla \mathfrak{B}_p^\mu + \frac{\hbar^2}{2(1-\mu)m}\nabla\left(\partial_\mu S_v^P \partial_\mu S_v^P\right) \tag{16}$$

And,

$$d_t\, S_p = \frac{e}{(1-\mu)m} S_p \times \mathfrak{B}_p \tag{17}$$

$$d_t\, S_e = -\frac{e}{\mu m} S_e \times \mathfrak{B}_e \tag{18}$$

Adding Eqs. (15), (16), and considering $p_e + p_p \approx p_F = \frac{(3\pi^2)^{2/3}\hbar^2}{5m\mu(1-\mu)} n^{5/3} \equiv \eta_0 n^{5/3}$, and using total spin currents density, we obtain the equation of evolution for u,

$$d_t\bar{u} + \frac{\eta_0}{m}\nabla n^{2/3} - \frac{\hbar^2}{2m^2\mu(1-\mu)}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) = \frac{1}{n}(\mathcal{S}\times B) + \frac{e\hbar}{2m^2c}\left\{\frac{1}{(1-\mu)} S^p \times \left[\nabla \times \left(B + \frac{\hbar c}{2e}\nabla\times(n\nabla\times S_p)\right)\right] + \frac{1}{\mu} S^e \times \left[\nabla\times\left(B - \frac{\hbar c}{2e}\nabla\times(n\nabla\times S_e)\right)\right]\right\} + \frac{\hbar^2}{2m^2}\left[\frac{1}{(1-\mu)}\nabla\left(\partial_\mu S_v^p \partial_\mu S_v^p\right) + \frac{1}{\mu}\nabla\left(\partial_\mu S_v^e \partial_\mu S_v^e\right)\right] \tag{19}$$

Or,

$$d_t\bar{u} = \frac{1}{n}(\mathcal{S}\times B) - \frac{\eta_0}{m}\nabla n^{2/3} + \frac{\hbar^2}{2m^2\mu(1-\mu)}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) + \frac{e\hbar}{2m^2c}\frac{1}{(1-\mu)\mu}\bar{S}\times(\nabla\times B) + \frac{e\hbar}{2m^2c}\left\{\frac{1}{(1-\mu)} S^p \times\nabla\times\left[\frac{\hbar c}{2en}\nabla\times(n\nabla\times S_p) + -\frac{1}{\mu}S^e\times\nabla\times\left(\frac{\hbar c}{2en}\nabla\times(n\nabla\times S_e)\right)\right]\right\} + \nabla\left[\mu(1-\mu)\frac{m^2c^2}{2n^2e^2}\mathcal{S}^2\right] \tag{20}$$

Where, we used the multiplication properties of the operators. Finally, it can obtain,

$$d_t\bar{u} = \frac{1}{n}(\mathcal{S}\times B) - \frac{\eta_0}{m}\nabla n^{2/3} + \frac{\hbar^2}{2m^2\mu(1-\mu)}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) + \frac{e\hbar}{2mc}\frac{1}{(1-\mu)\mu}\bar{S}\times(\nabla\times B) + \mu(1-\mu)\frac{m^2c^2}{2n^2e^2}[\mathcal{S}\times\nabla\times\mathcal{S}] + \frac{e\hbar}{2mc}\left[\frac{1}{(1-\mu)}S_\mu^p\nabla\times\left(\frac{\hbar c}{2en}\nabla\times(n\nabla\times S_p)\right) - \frac{1}{\mu}S_\mu^e\nabla\times\left(-\frac{\hbar c}{2en}\nabla\times(n\nabla\times S_e)\right)\right] + \nabla\left[\mu(1-\mu)\frac{m^2c^2}{2n^2e^2}\mathcal{S}^2\right] \tag{21}$$

Or,

$$d_t\bar{u} = \frac{1}{n}(\mathcal{S}\times B) + \frac{e\hbar}{2mc}\frac{1}{(1-\mu)\mu}\bar{S}\times(\nabla\times B) + \mu(1-\mu)\frac{m^2}{2n^2e^2}[\mathcal{S}\times(\nabla\times\mathcal{S})] + \nabla\left[\mu(1-\mu)\frac{m^2}{2n^2e^2}\mathcal{S}^2\right] - \frac{\eta_0}{m}\nabla n^{2/3} + \frac{\hbar^2}{2m^2\mu(1-\mu)}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) \tag{22}$$

On the other hand, using Eqs. (6), (10), and (11), we can write,

$$E = -\partial_t A - \nabla\varphi = -u_e \times B + \frac{\hbar^2}{2e\mu m}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) - \frac{\mu m}{e} d_t u_e - \frac{\eta_0}{e}\nabla p_e - \frac{\hbar e}{2\mu m^2} S_\mu^e \nabla\mathfrak{B}_e^\mu + \frac{\hbar^2}{2\mu m^2}\nabla\left(\partial_\mu S_v^e \partial_\mu S_v^e\right) \tag{23}$$

Calculating curl on both sides of Eq. (14), all terms resulting from the gradient become zero. So, the evolution of the magnetic field become,

$$\partial_t B = \nabla\times[\bar{u}\times B] + \frac{\mu m}{e} d_t[\nabla\times\bar{u}] + \frac{e\hbar}{2m^2}\nabla S\times\nabla B \tag{24}$$

Adding two Eqs. (7) and (9), we obtain,

$$d_t\bar{S} = \frac{e}{m}\,\bar{S}\times\left[B + \frac{\hbar}{2ne}\nabla(n\nabla.\bar{S})\right] \tag{25}$$

Let us consider the equilibrium state where $B = B\hat{z}$, $\langle u\rangle = 0$, $k = k(0, \sin\varphi, \cos\varphi)$, where φ is the angle between wave vector and magnetic field.

For Parallel propagation, $k \parallel B$, the linear perturbations around the equilibrium state. Using Eqs. (8), (9), (10), (14), (15), (16), (17), and (18), we can write,

$$\begin{bmatrix} 0 & 0 & B & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & -\eta & A_5 & 0 & 0 & 0 & B \\ 0 & 0 & 0 & 0 & 1 & 0 & B & 0 \\ 0 & 0 & 0 & 1 & A_1 B & -A_3 & 1 & 0 \\ 0 & 0 & 0 & -A_2 & 0 & B & 0 & 0 \\ 0 & 0 & 0 & A_1 B & A_2 & 1 & A_3 & 0 \\ -iB & -A_4 & 0 & 0 & e/mc & 0 & 0 & 0 \\ A_4 & -iB & 0 & -e/mc & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} S_x \\ S_y \\ n \\ b_y \\ b_x \\ u_n \\ u_y \\ u_z \end{bmatrix} = 0 \tag{26}$$

Where, $A_1 \equiv \left[1 + \mu(1-\mu)\frac{m^2k^2}{e^2}\right]$, $A_2 \equiv k\cos\varphi = \frac{i(1-\mu)m}{e}$, $A_3 \equiv i\frac{\mu m\omega}{e}$, $A_4 \equiv \frac{e}{\mu m}\left(1 + \frac{\hbar k^2}{2e}\right)$, $A_5 = \left(1 + \frac{\hbar e k}{2\mu m^2}\right)\sin\varphi \equiv 0$, $\eta = \frac{1}{\mu(1-\mu)}\left(\frac{5}{3}\frac{\eta_0}{m} + \frac{1}{4}\frac{\hbar^2}{m^2}k^2\right)$, $\eta_0 = \frac{(3\pi^2)^{2/3}\hbar^2}{5m\mu(1-\mu)}$ and $B \equiv \omega/k$. The spin effect appears only in the four elements $\begin{bmatrix} -i\,\omega/k & -A_4 \\ A_4 & -i\,\omega/k \end{bmatrix}$ and does not appear in the rest of them. Use the properties of determinants, we have [21, 22],

$$\det(8\times 8)=\det(6\times 6)\ \times \det\begin{vmatrix}-iB & -A_4\\ A_4 & -iB\end{vmatrix}=0 \quad (27)$$

Solving $\det(6\times 6)=0$, normal modes without spin can obtain, and solving the remaining determinant, normal modes related to spin can obtain. The frequency of this spin mode is equal to $\omega\equiv\omega_{ws}$, so,

$$\omega_{ws}=\frac{\hbar e}{2\mu m}k^3+\frac{e}{\mu m}k \quad (28)$$

For electron-proton state, Eq. (28) can rewrite as,

$$\omega_{ws}=8.9\times 10^{-24}k^3+1.8\times 10^{11}k$$

Parallel propagation has an acoustic mode with the characteristic equation, $\begin{bmatrix}\omega/k & -1\\ -\eta & \omega/k\end{bmatrix}\begin{bmatrix}n\\ u_z\end{bmatrix}=0$, or $\begin{vmatrix}\omega/k & -1\\ -\eta & \omega/k\end{vmatrix}=0$. It's solution is $\omega^2=\eta k^2=\frac{1}{\mu(1-\mu)}\left(\frac{5}{3}\frac{\eta_0}{m}+\frac{1}{4}\frac{\hbar^2}{m^2}k^2\right)k^2$ or $\omega^2=7.4\times 10^{30}k^2+3.7\times 10^{-9}k^4$. The second term is explicitly related to $\hbar$ and is therefore derived from quantum properties and becomes dominant for $k\geq 10^{18}$. For Perpendicular propagation, $k\perp B$, we can write,

$$\begin{bmatrix}0&0&B&0&0&0&0&-1\\0&0&-\eta&A_5&0&0&0&B\\0&0&0&0&0&0&B&0\\0&0&0&0&A_1B&-A_3&0&0\\0&0&0&-A_2&0&B&0&0\\0&0&0&A_1B&A_2&0&A_3&1\\-iB&-A_4&0&0&e/mc&0&0&0\\A_4&-iB&0&-e/mc&0&0&0&0\end{bmatrix}\begin{bmatrix}S_x\\S_y\\n\\b_z\\b_x\\u_z\\u_x\\u_y\end{bmatrix}=0 \quad (29)$$

Where $A_1\equiv\left[1+\mu(1-\mu)\frac{m^2k^2}{e^2}\right]$, $A_2\equiv\frac{i(1-\mu)m}{e}k\cos\varphi=0$, $A_3\equiv i\frac{\mu m\omega}{e}$, $A_4\equiv\frac{e}{\mu m}\left(1+\frac{\hbar k^2}{2e}\right)$, $A_5=\left(1+\frac{\hbar e k}{2\mu m^2}\right)\equiv 0$, $\eta=\frac{1}{\mu(1-\mu)}\left(\frac{5}{3}\frac{\eta_0}{m}+\frac{1}{4}\frac{\hbar^2}{m^2}k^2\right)$, $\eta_0=\frac{(3\pi^2)^{2/3}\hbar^2}{5m\mu(1-\mu)}$ and $B\equiv\omega/k$. We must consider the remaining elements corresponding to $(u_x, b_x, u_z,$ and $b_z)$. The result is,

$$\left(\frac{\omega}{k}\right)^4-\frac{\left(1+\frac{e\hbar k}{2\mu m^2}\right)}{1+\mu(1-\mu)\frac{m^2}{e^2}k^2}\left(\frac{\omega}{k}\right)^2-\frac{1}{\mu(1-\mu)}\left(\frac{5}{3}\frac{\eta_0}{m}+\frac{1}{4}\frac{\hbar^2}{m^2}k^2\right)=0 \quad (30)$$

Or,

$$\omega^4-\frac{\left(1+\frac{e\hbar k}{2\mu m^2}\right)k^2}{1+\mu(1-\mu)\frac{m^2}{e^2}k^2}\omega^2-\frac{k^4}{\mu(1-\mu)}\left(\frac{5}{3}\frac{\eta_0}{m}+\frac{1}{4}\frac{\hbar^2}{m^2}k^2\right)=0 \quad (31)$$

3. Electrons as an Impurity

We study the effect of the motion of an electron on a one-dimensional grid of ions as an impurity and its effect on ground state energy.

3.1. Hamiltonian and L. L.P Transformation

We assume that the density of ions is much higher than that of electrons. Again, for simplicity, we assume our system is electron-proton. At any given length $\mathcal{L}_0$, there is one electron impurity as a Fermi polaron. We study its effect on ground state energy. By eliminating the degree of freedom of the impurity, we replace a generalized Hamiltonian of only protons (impurity removal) with the system's Hamiltonian. We use the Variation-Method [23] and the Lee-Low-Pines (LLP) transformation [24-26] for the system's ground state. The Hamiltonian of the entire system is [27-29],

$$H=H_e+H_p \quad (32)$$

$$H=\frac{p_p^2}{2m}+\sum_{i=1}^{N}\frac{p_{ei}^2}{2\mu m}+g\sum_{i=1}^{N}\delta(x_{ei}-x_p) \quad (33)$$

Where, g shows the strength of the interaction of each proton with the electron impurity. or,

$$H_e=\int_0^L dx\,\psi_e^\dagger(x)\left(-\frac{1}{2\mu m}\frac{\partial^2}{\partial x^2}\right)\psi_e(x) \quad (34)$$

and,

$$H_p=\int_0^L dx\left[\psi_p^\dagger(x)\left(-\frac{1}{2\mu m}\frac{\partial^2}{\partial x^2}\right)\psi_p(x)+g\psi_e^\dagger(x)\psi_p^\dagger(x)\psi_p(x)\psi_e(x)\right] \quad (35)$$

Where L is the total length of the grid. The system stays in the polaron state, defined as the minimum energy state at a given total momentum $Q=p_p+\sum_{i=1}^{N}p_{ei}$ or $\sum_{i=1}^{N+1}p_i$ where $p_{N+1}\equiv p_p$. For simplicity, we assume the protons have a constant mean distance $\bar{d}=1, n=\mathcal{L}_0/\bar{d}$. In this case, the share of spin in the system Hamiltonian is [30-32],

$$H=-\xi_k\sum_\beta C_{\beta,e}^\dagger\ C_{\alpha,e}+g\sum_\beta C_{\beta,p}^\dagger C_{\beta,p}C_{\beta,e}^\dagger C_{\beta,e}-\mu\sum_\beta C_{\beta,p}^\dagger C_{\beta,p} \quad (36)$$

Where, $C_{\beta p}^\dagger$ and $C_{\beta,p}$ is the creation and annihilation operators for an electron at x_β, $C_{\beta,e}^\dagger$, and $C_{\beta,e}$ for proton at x_α, μ is chemical potential and $\xi_k=\frac{p^2}{2\mu m}$. The operator $C_{\beta,ep}^\dagger$ creates a proton with momentum p and spin index p. The system Hamiltonian in Eq. (36) can also write as,

$$H=-2\xi_k C_{\beta,e}^\dagger\ C_{\alpha,p}+\sum_\beta(\varepsilon_\beta-\mu)C_{\beta,e}^\dagger C_{\beta,e}+\frac{g}{N}\sum_{\mu,\beta,\beta'}C_{\mu,p}^\dagger\ C_{\mu,p}e^{i(k'-k).x_\mu}C_{\beta,e}^\dagger C_{\beta',e} \quad (37)$$

Where $C_{k,p}=\frac{1}{\sqrt{N}}\sum_\mu e^{-i\,k.x_\mu}C_{\mu,p}$ and $C_{k,e}^\dagger=\frac{1}{\sqrt{N}}\sum_\mu e^{i\,k.x_\mu}\ C_{\mu,e}$ are the electron and proton operators in the momentum space, and N is the number of sites in the grid. The parameter g is also called the strength of interaction with the scattering length $\alpha\equiv\frac{2\pi}{d}$. To study the physics of Polaron, we assume there is only one electron impurity particle. We want to eliminate the degree of freedom of the impurity by the unitary transformation $U=\exp[-ip.x]$. This transformation operator

LLP for the annihilation operator C_{pe} is,

$$UC_{pe}U^{-1} = C_{pe}e^{-p.x} \tag{38}$$

Where, $p = \sum_{\beta} x_{\beta}^{-1} C_{\beta,p}^{\dagger} C_{\beta,p}$ is the operator of the total momentum of the protons, x_{β}^{-1} is the inverse grid vector, and $X = \sum_{\mu} x_{\beta} C_{\mu p}^{\dagger} C_{\mu p}$ is the operator of electron impurity coordinates. The above unitary transformation equation is called Lee- Low- Pins (LLP). This transformation introduces to separate the degrees of freedom of the electron and the proton base in a Bose medium of phonons [33, 34]. We transform the spin-electron and spin-proton operators C_{ke} and C_{kp} to $U^{\dagger}C_{ke}U = e^{-ik.x}C_{ke}$ and $U^{\dagger}C_{kp}U = e^{-ik.x}C_{kp}$. As a result, the share of spins in Hamiltonian is [35, 36],

$$H = \sum_{\mu} C_{\mu e}^{\dagger} C_{\mu e} \left[\frac{\xi_k}{2} \sum_n e^{-i(\mathbf{p}-\mathcal{P}).\mathbf{nd}}\right] + \sum_{\mu}(\varepsilon_{\mu} - \mu) C_{\mu p}^{\dagger} C_{\mu p} + \sum_{\mu'} C_{\mu' e}^{\dagger} C_{\mu' e} \left(\frac{g}{N} \sum_{\beta,\mu} C_{\beta p}^{\dagger} C_{\mu p}\right) \tag{39}$$

Where d is the distance vector between two adjacent protons, as can be seen, this variation method disrupts the conservation of the system's total momentum. Part of the protons' momentum is transferred to the electron, eliminating the degree of freedom of the impurity particle. Therefore, for a given total momentum p_{tot}, the contribution of only the protons Hamiltonian can be written as,

$$H_{p,e} = \sum_{\beta}(\varepsilon_{\beta} - \mu) C_{\beta,p}^{\dagger} C_{\beta,p} + \frac{g}{N} \sum_{\beta,\mu} C_{\beta,p}^{\dagger} C_{\mu,p} + \frac{\xi_k}{2} \sum_n e^{-i(p-\mathcal{P}).nd} \tag{40}$$

Now, we create a wave function as $|\psi\rangle = U|0\rangle_p$ to find the approximate ground state of the original Hamiltonian. Then by using the impurity state and the inverse of the transformation, we get the original state of the initial Hamiltonian with a constant momentum.

By adding one electron impurity and inverse transformation, the eigenstate of the primary Hamiltonian state in Eq. (37) can express as a new state. This situation is similar to the existence of a hole at the top of the Fermi electrons. According to the variation method, the ground state of Hamilton H can write in terms of the imaginary time evolution of the wave function $|\psi(\tau)\rangle = e^{-H\tau}|\psi(0)\rangle$ where $\psi(0)\rangle$ shows the initial state. The ground state of Eq. (39) can be written in terms of a wave function as the state $|\tilde{\psi}(\tau)\rangle$ holds in the following differential equation,

$$d_{\tau}|\psi(\tau)\rangle = -[H - \langle H\rangle]|\psi(\tau)\rangle \tag{41}$$

And for $\tilde{\psi}(\tau)$,

$$d_{\tau}|\tilde{\psi}\rangle = -[H - \langle H\rangle]|\tilde{\psi}\rangle = -U\left[\left(C_{\boldsymbol{ke}}^{\dagger}|0\rangle_e\right) \otimes U \emptyset^T U_m^T E'_m U_m \emptyset |0\rangle_p\right] \tag{42}$$

Where $\langle H\rangle = \langle\psi(\tau)|H|\psi(\tau)\rangle$ and $\bar{E} = \langle\tilde{\psi}|H|\tilde{\psi}\rangle = \langle\psi|H_{pe}|\psi\rangle$ are the average energy of wave function $\psi(\tau)$ and $\tilde{\psi}(\tau)$. By calculating the variation energy $E = \langle\psi|H_{\boldsymbol{pe}}|\psi\rangle$, using the relations $C_{\mathbf{k},p}^{\dagger} = \frac{1}{2}(b_{1,\boldsymbol{k}} - ib_{2,\boldsymbol{k}})$, $C_{\boldsymbol{k},p} = \frac{1}{2}(b_{1,\boldsymbol{k}} + ib_{2,\boldsymbol{k}})$ and some mathematical calculation, we obtain,

$$E = \langle\psi|H_{pe}|\psi\rangle = \frac{1}{2}\sum_{\mu} \varepsilon_{\mu} - \frac{1}{2}N\mu + \frac{g}{2} + \frac{1}{4}\sum_{\mu,\beta}(H_0)_{\mu,\beta} (Q_m)_{\mu,\beta} + -\frac{\xi_k}{2}\sum_{v} e^{-ik.v}\left(\langle\psi|e^{i\mathcal{P}.x}|\psi\rangle\right) - \frac{1}{4}\mu \sum_{\mu,\beta} \mathcal{F}_{\mu,\beta} (Q_m)_{\mu,\beta} \tag{43}$$

where, the function $\mathcal{F}$ is defined as $\mathcal{F} \equiv \begin{bmatrix} 0 & \mathbb{I}_N \\ \mathbb{I}_N & 0 \end{bmatrix}$, Eq. (43) has the minimum energy as,

$$E_{min} = 1 + \frac{1}{\pi\alpha} - \frac{1+\alpha^2-\delta^2}{2\alpha}\left(\beta + \frac{2}{\alpha}\right) + \delta\varphi \tag{44}$$

where, $\beta = \frac{1}{\alpha\pi}[arctan(\alpha - \delta) + arctan(\alpha + \delta)]$ and $\varphi = \frac{1}{2\pi\alpha^2} ln \frac{1+(\alpha-\delta)^2}{1+(\alpha+\delta)^2}$.

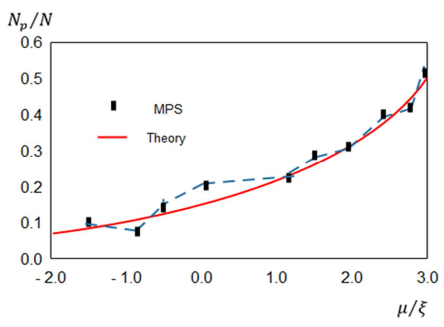

Figure 1. *The plot shows the number of protons and one Fermi polaron in a 1D grid as the chemical potential. The results obtained with theory and the MPS (matrix product state) method is in perfect agreement. The parameters used in this figure are* $g/\xi_k = 3$, $p_{tot} = 0$, *and* $N = 80$.

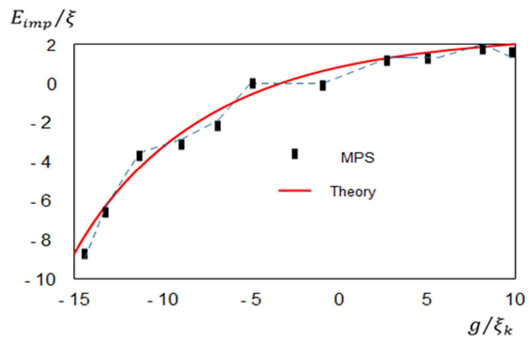

Figure 2. *The ground state energy of an electron in a 1D grid as the interaction strength* g/ξ, *with* $N_p = 75$, $p_{tot} = 0$, *and* $N = 100$.

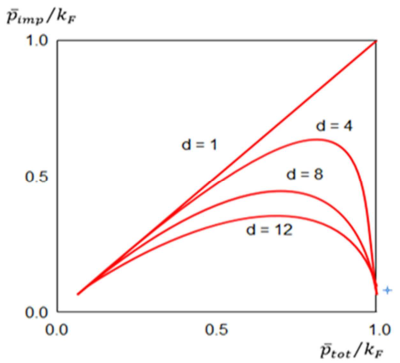

Figure 3. *Plot shows the average momentum $\bar{p}_{imp}$ of the impurity as a function of the total momentum $\bar{p}_{total}$ for different values d (ground state). The top lines to bottom are for $d = 1, 4, 8, 12$, respectively.*

3.2. Analysis

The state $p_{tot} = 0$ is completely consistent with what was obtained using the MPS method without determining the total momentum p_{tot}. It shows that the ground state has a total momentum magnitude of zero. We obtain the Polaron energy as $E_p(p_{tot}, g) = E(p_{tot}, g) - E(p_{tot} = 0, g = 0)$. Note that the energy of the non-interactive system $E(p_{tot} = 0, g = 0)$ can be calculated exactly. In Figure 1, it can be seen that the ground state energy changes smoothly in terms of interaction. At the infinitely large repulsion limit $g \to +\infty$, it creates one electron impurity like a hard and effective wall, which results in energy of $E_p \to 2\xi_k$. In contrast to the sizeable attractive interaction limit $g \to -\infty$, the electron impurity pairs firmly with a proton. Therefore, energy tends to have a limit value of $E_e \to g + 2\xi_k$.

4. Conclusion

In this paper, a set of two-fluid electron-ion plasmas equations based on the hydrodynamic model was written and briefly discussed its various terms and the role of spin in spin-spin and spin-magnetic field interactions. These equations showed what dispersion relations to expect if turbulence occurs in equilibrium plasma. Here, we take a path for discussion that differs from the typical path taken in other articles and obtain new dispersion equations. We also examined the limits of weak and strong magnetic fields and the effect of spin polarization on ripple waves in plasma. Again, assuming that the external magnetic field is weak, we obtained a quasi-sound wave due to differences in the distribution of electron-proton states. We found that the spin-current evolution in magnetized plasma creates a new dispersion mode. We also showed that we have only the fast mode in the direction perpendicular to the waves' propagation direction. The speed of this mode is equal to the speed of the mode in the parallel direction plus an additional term that depends on the system's characteristics. For high-density plasma, this correction is negligible. However, for very low densities and weak magnetic fields, this effect is significant.

We have studied the polaron properties in a homogeneous Fermi gas of electron in a proton base by using a variation method in the one-dimension grid (Fermi gas in one-dimension). We assume the density of protons is much higher than that of electrons. There is one electron impurity in the one-dimensional grid of protons at any given length L_0 as a Fermi polaron. We study its effect on ground state energy. By eliminating the degree of freedom of the impurity, we replace a generalized Hamiltonian of only protons (impurity removal and consider the proton grid as it is in a Bose medium of phonons. The ground state was obtained for a total fixed momentum via imaginary time evolution for the various parameters. Also, by using the system Hamiltonian, we obtained the minimum energy and energy of the system.

Author's Contribution

Each author's contribution to the composition of the article is the same.

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